Non-sequential finite automata

Szilárd Fazekas **Robert Mercaş** Luca Prigioniero

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Loughborough University

Automata with non-sequential processing modes

Restarting automata [Jančar et al., 1995]

Revolving input automata [Bordihn et al., 2005]

Automata with translucent letters [Nagy and Otto, 2011]

Jumping automata [Meduna and Zemek, 2012]

(Right) One-way jumping automata [Chigahara et al., 2016]

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Presented as $M = (Q, \Sigma, R, s, F)$ (just as a regular DFA)

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Accepts permutation-closed semilinear languages (incomparable to REG)

Right one-way jumping finite automaton [Chigahara et al., 2016]

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Elements of R are transition rules $\mathbf{p}a \to \mathbf{q} \in R$

Configurations of M are strings in $Q\Sigma^*$

Working

$x, y \in \Sigma^*$, $a \in \Sigma$, $\mathbf{p}, \mathbf{q} \in Q$ and $\mathbf{p}a \to \mathbf{q} \in R$

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The *right one-way jumping relation* $\circlearrowright_R$ over $Q\Sigma^*$, ($\circlearrowright_R$DFA M jumps from configuration $\mathbf{p}xay$ to $\mathbf{q}yx$):

$$\mathbf{p}xay \circlearrowright \mathbf{q}yx$$

if $x \in \{\Sigma \setminus \Sigma_p\}^*$, where

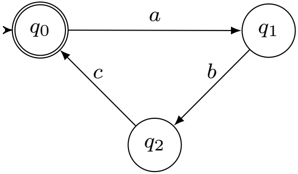
$$\Sigma_p = \{b \in \Sigma \mid (\mathbf{p}, b, \mathbf{p}') \in R \text{ for some } \mathbf{p}' \in Q\}$$

Example

Consider the $\circlearrowright_R$DFA M given in the transition graph below

Accepted language is $\{w \in \{a, b, c\}^* \mid |w|_a = |w|_b = |w|_c\}$

$$\mathbf{q}_0 acbcab \circlearrowright_R \mathbf{q}_1 bcabc \circlearrowright_R \mathbf{q}_2 cabc \circlearrowright_R \mathbf{q}_0 abc \circlearrowright_R \mathbf{q}_1 bc \circlearrowright_R \mathbf{q}_2 c \circlearrowright_R \mathbf{q}_0$$



Accepting power

$\mathbf{REG} \subsetneq \circlearrowright_R\mathbf{DFA}$

$\mathbf{CF}$ and $\circlearrowright_R\mathbf{DFA}$ are incomparable

$\circlearrowright_R\mathbf{DFA} \subsetneq \mathbf{CS}$

$\circlearrowright_R\mathbf{DFA} \subseteq \mathrm{DTIME}(n^2)$

Overview

No unique minimal $\circlearrowright_R$DFA

Different minimal $\circlearrowright_R$DFAs accepting language $\{w \in \{a,b\}^* \mid |w|_b > 0\}$

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with End Marker *added to* $\circlearrowright_R$*DFA we have closure on complement*

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Good decidability properties [Beier and Holzer, 2020]

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$\circlearrowright_R$ mode studied with more general machine models: NFA, 2DFA, PDA, LBA [Fazekas et al., 2019, Beier and Holzer, 2022, Fazekas et al., 2021]

Jumps and sweeps

A $\circlearrowright_R$DFA transition from $\mathbf{p}ax$ to $\mathbf{q}y$, denoted $\mathbf{p}ax \vdash \mathbf{q}y$:

(i) $\mathbf{p}ax \Rightarrow \mathbf{q}y$, where $x = y$ and $\mathbf{p}a \to \mathbf{q} \in R$ (*sequential trans.*)

(ii) $\mathbf{p}ax \circlearrowright \mathbf{p}xa$, when $a \in \Sigma \setminus \Sigma_p$, $\mathbf{p} = \mathbf{q}$ and $xa = y$ (*a jump*)

w accepted by M if $\mathbf{s}w \models^* \mathbf{f}$, and $L(M) = \{x \in \Sigma^* \mid \exists \mathbf{f} \in F : \mathbf{s}x \models^* \mathbf{f}\}$

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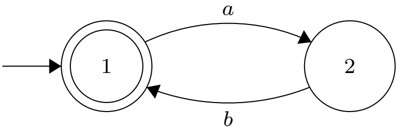
Deficient states: $\mathbf{p} \in Q$ is S-deficient if for every $a \in S \subset \Sigma$ and any $\mathbf{q} \in Q$, we have $\mathbf{p}a \to \mathbf{q} \notin R$



$\circlearrowright_R$DFA accepting $\{w \mid |w|_a = |w|_b\}$

Sweeps:

position :	0	1	2	3	4	5	6	7
input	**a**	**a**	**a**	**a**	**b**	**b**	**b**	**b**
after sweep 1	ε	**a**	**a**	**a**	ε	**b**	**b**	**b**
after sweep 2	ε	ε	**a**	**a**	ε	ε	**b**	**b**
after sweep 3	ε	ε	ε	**a**	ε	ε	ε	**b**
after sweep 4	ε	ε	ε	ε	ε	ε	ε	ε

Computation table for a^4b^4

Sweep complexity [Fazekas et al., 2022]

The *sweep complexity* of an automaton M is $sc_M(n)$ is the maximum number of sweeps M makes on processing inputs $w \in L(M)$ of length n

SWEEP$(f(n))$ class of languages accepted by $\circlearrowright_R$DFA with

$$sc_M(n) = \mathcal{O}(f(n))$$

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Theorem ([Fazekas and Yamamura, 2016])

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Previously known languages accepted by $\circlearrowright_R$DFA had sweep complexity either constant or linear (which is also the upper bound)

Questions regarding sweep complexity [Fazekas et al., 2022]

1. Is the language of each machine with $\omega(1)$ complexity non-regular? 'YES'
2. Is there a machine with sweep complexity between constant and linear, that is, $\omega(1)$ and $o(n)$? 'NO'
3. Is there a *language* with sweep complexity between constant and linear, that is, all machines accepting it have superconstant complexity and at least one has sublinear? 'NO'
4. Is there an upper bound in terms of sweep complexity on machines accepting regular languages? 'YES'
5. Are machines less complex in the case of binary alphabets? 'YES'

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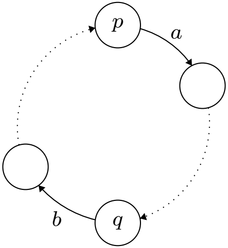
5. Are machines less complex in the case of binary alphabets? NO

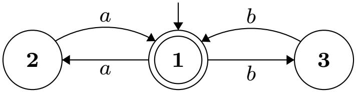
Necessary condition for $\omega(1)$ sweep complexity

Lemma ([Fazekas et al., 2022])

If a $\circlearrowright_R$DFA has superconstant sweep complexity, then it has two reachable and co-reachable states **p** *and* **q** *such that* **p** *is a-deficient,* **q** *is b-deficient, for some* $a, b \in \Sigma$ *with* $a \neq b$*, and* $\mathbf{p}buav \vdash^* \mathbf{q}av \vdash^* \mathbf{p}$*, for some* $u, v \in \Sigma^*$*.*



Logarithmic complexity



$$L(\mathcal{A}) = \{w \in \{a,b\}^* \mid |w|_a \equiv 0 \bmod 2, |w|_b \equiv 0 \bmod 2\}$$

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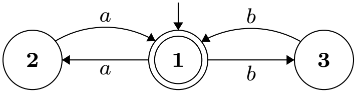

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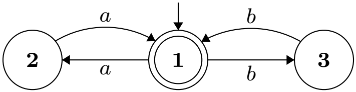

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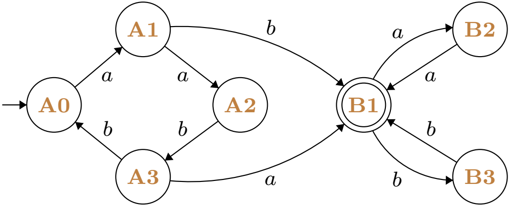
Proposition

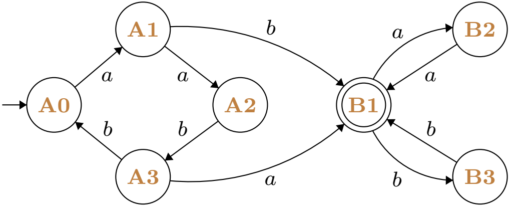
The sweep complexity of $\mathcal{A}$ *is* $\Theta(\log n)$.

Linear complexity



$$L(\mathcal{B}) = \{w \in \{a,b\}^* \mid |w|_a \equiv 1 \mod 2, |w|_b \equiv 1 \mod 2\}$$

Linear complexity



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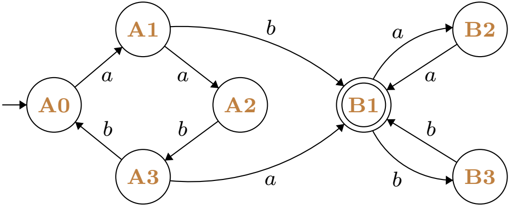

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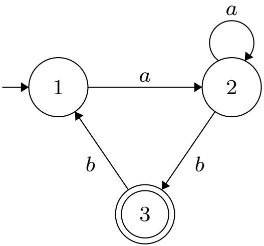
Non-REG language with sublinear sweep complexity



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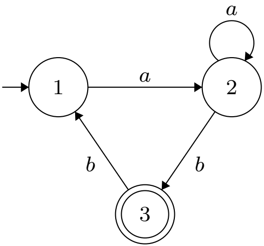

Lemma

The $\circlearrowleft_R$DFA $\mathcal{C}$ accepts a non-regular language.

Lemma

The sweep complexity of $\mathcal{C}$ is $\Theta(\log n)$.

$L(\mathcal{C})$ is not regular

Consider the morphism $\varphi : \{a,b\}^* \to \{a,b\}^*$ with $\varphi(a) = abab$, $\varphi(b) = b$

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(By induction) the last block of b's in $\varphi^n(ab)$ has length $n+1$, and is preceded by 2^n blocks of a's separated by blocks of b's

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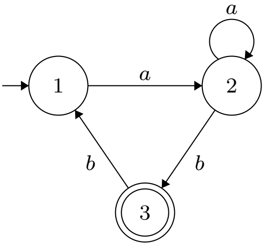

$$\mathbf{1}\varphi(ab) = \mathbf{1}ababb \Rightarrow^2 \mathbf{3}abb \circlearrowright a\mathbf{3}bb \Rightarrow a\mathbf{1}b \circlearrowright \mathbf{1}ab = \mathbf{1}\varphi^0(ab)$$

Complexity of $\mathcal{C}$ is $\Theta(\log n)$

$\mathcal{C}$ accepts $\varphi^k(ab)$ in $k+1$ sweeps and $|\varphi^k(ab)| \in \mathcal{O}(2^k)$
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Within a sweep

- each block of a's is fully processed if a letter is processed from them
- no two consecutive blocks of a can be jumped over
- the number of blocks of a's is reduced by at least half

so at most $\mathcal{O}(\log n)$ sweeps possible

Separating complexity classes

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Theorem

For any $f : \mathbb{N} \Rightarrow \mathbb{N}$ *with* $f(n) \in o(n)$ *we have* $SWEEP(f(n)) \subsetneq SWEEP(n)$.

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- one of the main features of these machines would be their size, as compared to regular DFA
- we could identify a minimum equivalent DFA when the expressed language is regular

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A $\circlearrowright_R$DFA accepts L_n by jumping the prefix of the input until reaching \$. Then stores x, and then, computing the jumped prefix, it compares each factor of length n with the string x stored in the finite control.

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A DFA accepts L_n by storing in its finite control the set S of n-length factors $x_1 x_2 \cdots x_k$ and, after reading \$, verifying that x is contained in S.

DFA requires double exponentially, in n, many states (to store S)

NFA to $\circlearrowright_R$DFA still exponential

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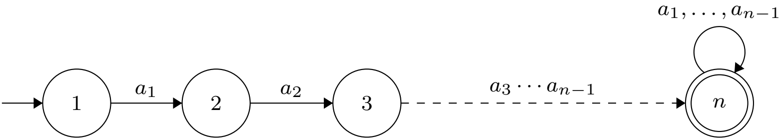
NFA to $\circlearrowright_R$DFA still exponential

The $\Theta(2^n)$ bound is straightforward

- Lower bound: classical example of n-th letter from the end being a
- Upper bound: follows from the fact that each DFA is an $\circlearrowright_R$DFA

$\circlearrowright_R$DFA to NFA ($\Sigma \in \mathcal{O}(n)$) still exponential



$\circlearrowright_R$DFA $\mathcal{T}$ from [Fazekas and Yamamura, 2016]

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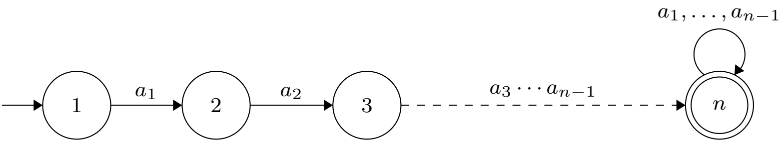

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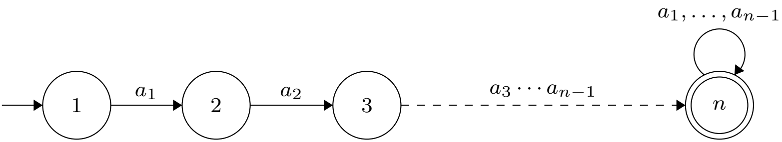

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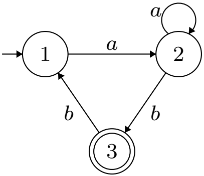
- $\circlearrowright_R$DFA $\mathcal{T}$ has n states, but any NFA needs 2^{n-1} states to keep track of which letters have already occurred in the input

$\circlearrowright_R$DFA to NFA ($|\Sigma| = 3$) still exponential



$\circlearrowright_R$DFA $\mathcal{C} \notin REG$ of sweep complexity $\Theta(\log n)$ [Fazekas and Mercaş, 2023]

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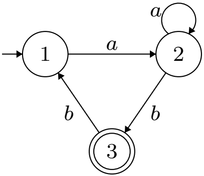

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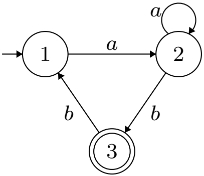

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- Words in $L(\mathcal{C})$ accepted in $\mathcal{O}(\log n)$ sweeps (some $\Omega(\log n)$ sweeps)
- Concatenate $n+1$ copies of $\mathcal{C}$ using a new symbol c to label the transitions between copies

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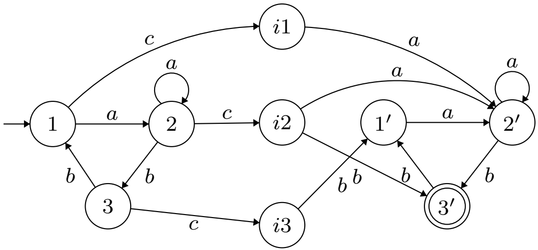

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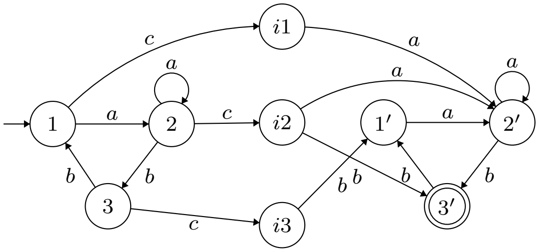

The resulting machine accepts $K \in REG$ and
$K \cap \{a, b\}^* b^{2n} c^n = \{w b^{2n} c^n \mid \text{number of blocks of } a\text{'s in } w \text{ is } \Omega(2^n)\}$

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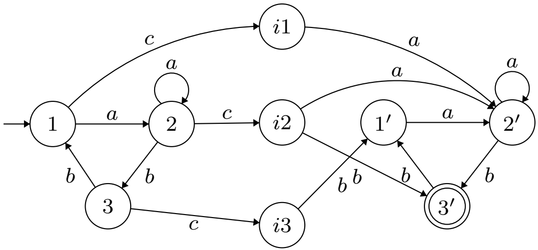

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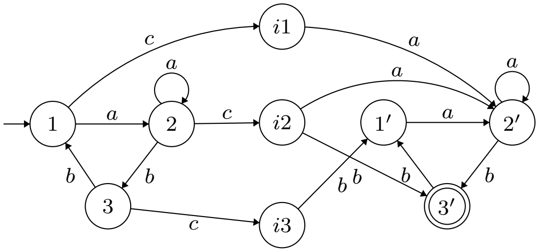

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An NFA accepting the language counts the a's and compares to count of b's, so it needs $\Omega(2^n)$ states (versus the $3(n+1)$ states of $\circlearrowright_R$DFA)

What is next?

- Are there machines with arbitrary (constructible) sublinear complexity ($\Theta(\log^k n)$ and $\Theta(n^\epsilon)$)?
- Is it decidable, given a machine or language and a function $f(n)$, whether the machine/language has $\Theta(f(n))$ sweep complexity?
- Solve the open problems regarding equivalence and regularity
- Nondeterminism...
- Smaller alphabet size for the descriptional complexity trade-offs

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Thank you!

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Extra I

Problem

Given an $\circlearrowright_R$*DFA* A *and word* w*, does there exist a* $u \in L(A)$ *such that* $w \leq_s u$*?*

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Theorem (Fazekas, Koss, Manea, M., Specht)

Given a $\circlearrowright_R$*DFA (or DFAwtl)* A *and a word* w *it is decidable (in NP time) whether there exists a word* $u \in L(A)$ *such that* $w \leq_s v$*?*

Exponential upper bound on the length of factors between letters of the subsequence

Done by iteratively inserting factors of bounded length (wrt number of states) between letters in each sweep, bottom up

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Theorem ([Beier and Holzer, 2020, Theorem 6])

Given a $\circlearrowright_R$DFA A and a word w, it is decidable in PSPACE whether there exists a $v \in L(A)$ such that (1) the word w is a prefix of v, (2) the word w is a sufix of v, (3) the word w is a factor of v, and (4) the word w is a subsequence of v.

Extra II

Theorem (Fazekas, Koss, Manea, M., Specht)

Given a $\circlearrowright_R$DFA A and a word w, both defined over a binary alphabet, it is decidable in NP time whether there exists a word $u \in L(A)$ such that $w \leq_s v$?

Assume that there exists $v \in L(A)$ with $w \leq_s v$

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For every $w \in \{a, b\}^k$, for every $v \in \{a, b\}^*$ with at least $2k$ blocks $w \leq_s v$

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When language accepted by A is finite, then the longest word accepted has at most as many letters as the number of states of A,
the length of the witness v is also upper bounded by $|A|$

Extra III

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Lemma ([Beier and Holzer, 2020, Lemma 14])

Let A be an $\circlearrowright_R$DFA and $L \subset \Sigma^*$ be a finite language. Then it is decidable whether (1) $L(A) \cap L = \emptyset$, (2) $L \subseteq L(A)$, (3) $L(A) \subseteq L$; and (4) $L = L(A)$.