Complexities (and results) for One-Way Jumping Automata

Big VMI event @FMI, 2023

S. Z. Fazekas   Robert Mercaș   O. Wu

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Loughborough University

Jumping Finite Automata

Jumping finite automata [Meduna and Zemek, 2012]

- process the input in arbitrary order

- can match number of letters

- accepts permutation-closed semilinear languages, so incomparable to REG [Fernau et al., 2015]

Right one-way jumping finite automaton ($\circlearrowright_R$DFA) [Chigahara et al., 2016]

$M = (Q, \Sigma, R, s, F)$, as in a DFA.

Elements of R are transition rules $\mathbf{p}a \to \mathbf{q} \in R$

Configurations of M are strings in $Q\Sigma^*$.
$x, y \in \Sigma^*$, $a \in \Sigma$, $\mathbf{p}, \mathbf{q} \in Q$ and $\mathbf{p}a \to \mathbf{q} \in R$.

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The *right one-way jumping relation* $\circlearrowright_R$ over $Q\Sigma^*$, ($\circlearrowright_R$DFA M jumps from configuration $\mathbf{p}xay$ to $\mathbf{q}yx$):

$$\mathbf{p}xay \circlearrowright \mathbf{q}yx$$

if $x \in \{\Sigma \setminus \Sigma_p\}^*$, where

$$\Sigma_p = \{b \in \Sigma \mid (\mathbf{p}, b, \mathbf{p}') \in R \text{ for some } \mathbf{p}' \in Q\}.$$

Example

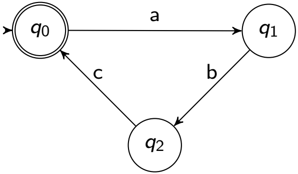
Let M be a $\circlearrowright_R$DFA given by

$$M = (\{\mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2\}, \{a, b, c\}, R, \mathbf{q}_0, \{\mathbf{q}_0\}),$$

where R consists of the rules $\mathbf{q}_0 a \to \mathbf{q}_1$, $\mathbf{q}_1 b \to \mathbf{q}_2$ and $\mathbf{q}_2 c \to \mathbf{q}_0$.

Accepted language is $\{w \in \{a, b, c\}^* \mid |w|_a = |w|_b = |w|_c\}$

$$\mathbf{q}_0 acbcab \circlearrowright_R \mathbf{q}_1 bcabc \circlearrowright_R \mathbf{q}_2 cabc \circlearrowright_R \mathbf{q}_0 abc \circlearrowright_R \mathbf{q}_1 bc \circlearrowright_R \mathbf{q}_2 c \circlearrowright_R \mathbf{q}_0$$



(Classic) example of $\circlearrowright_R$**DFA**

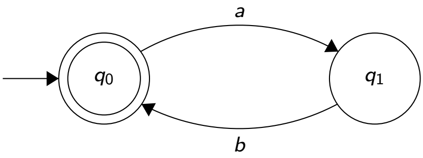


Figure: DFA for the regular language $(ab)^*$ and $\circlearrowright_R$**DFA** for the non-regular language $L(\mathcal{M}_{ab})\{w \mid |w|_a = |w|_b\}$

We have strict inclusion of languages, among **DFA**, $\circlearrowright_R$**DFA**, and **JFA**

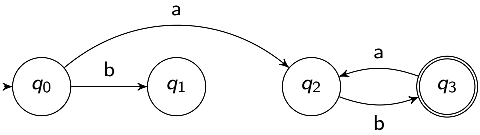


Figure: The automaton A with $L_D(A) \subsetneq L_R(A) \subsetneq L_J(A)$ [Beier and Holzer, 2018a]

$$L_D(A) = (ab)^+$$

$$L_R(A) = \{a\,w\,b \mid |w|_a = |w|_b\}$$

$$L_J(A) = \{a\,w \mid |w|_a + 1 = |w|_b\}$$

Who's who

- **REG** $\subsetneq$ $\circlearrowright_R$**DFA**.
- **CF** and $\circlearrowright_R$**DFA** are incomparable.
- $\circlearrowright_R$**DFA** $\subsetneq$ **CS**.

Not a very nice class (closure wise)

$\circlearrowright_R$**DFA** is not closed under "anything" ($\cup$, $\cap$, $\cdot$, $\overline{L}$, reversal, intersect. with reg. lang., homomorphisms, etc.)

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$\circlearrowright_R$ processing mode studied with more general machine models: NFA, PDA, LBA, 2Way Jumping [Fazekas et al., 2019, Beier and Holzer, 2019, Fazekas et al., 2021]

Closed under	**REG**	$\circlearrowright_R$**DFA**	**JFA**
Union	yes	no	yes
Union with reg	yes	no	no
Intersection	yes	no	yes
Intersection with reg	yes	no	no
Complementation	yes	no	yes
Reversal	yes	no	yes
Concatenation	yes	no	no
Right conc. with reg	yes	no	no
Left conc. with reg	yes	yes	no
Kleene star or plus	yes	no	no
Homomorphism	yes	no	no
Inv. homomorphism	yes	no	no
Substitution	yes	no	no
Permutation	no	no	yes

Table: The Closure Properties of **REG**, $\circlearrowright_R$**DFA** and **JFA** [Beier and Holzer, 2018a]

1. Preliminaries
2. Model restrictions
3. Not again restrictions!
4. Conclusions

Some open questions

How do you get **REG**?

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Theorem ([Fazekas and Yamamura, 2016])

For any $\circlearrowright_R$**DFA** $\mathcal{A} = (Q, \Sigma, R, \mathbf{s}, F)$ *and any constant* k*, the set of words accepted by* $\mathcal{A}$ *in at most* k *sweeps is regular.*

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All known classes of languages accepted by $\circlearrowright_R$**DFA** had sweep complexity either constant or linear (which is also the upper bound).

Basics on $\circlearrowright_R$DFA

A $\circlearrowright_R$**DFA** jumping transition from $\mathbf{p}ax$ to $\mathbf{q}y$, denoted $\mathbf{p}ax \models \mathbf{q}y$:

(i) $\mathbf{p}ax \Rightarrow \mathbf{q}y$, where $x = y$ and $\mathbf{p}a \to \mathbf{q} \in R$, as defined earlier, or

(ii) $\mathbf{p}ax \circlearrowright \mathbf{p}xa$, when $a \in \Sigma \setminus \Sigma_p, \mathbf{p} = \mathbf{q}$ and $xa = y$. (*a jump*)

w accepted by M if $\mathbf{s}w \models^* \mathbf{f}$, and $L(M) = \{x \in \Sigma^* \mid \exists \mathbf{f} \in F : \mathbf{s}x \models^* \mathbf{f}\}$.

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$\mathbf{p} \in Q$ is S-deficient if $\forall a \in S \subset \Sigma$ we have $\mathbf{p}a \to \mathbf{q} \notin R$ for every $\mathbf{q} \in Q$

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Sweeps:

position :	0	1	2	3	4	5	6	7
input	**a**	**a**	**a**	**a**	**b**	**b**	**b**	**b**
after sweep 1	ε	**a**	**a**	**a**	ε	**b**	**b**	**b**
after sweep 2	ε	ε	**a**	**a**	ε	ε	**b**	**b**
after sweep 3	ε	ε	ε	**a**	ε	ε	ε	**b**
after sweep 4	ε	ε	ε	ε	ε	ε	ε	ε

Figure: The computation table for a^4b^4 by $\mathcal{M}_{ab}$.

Definition of VMI for complexities [Fazekas et al., 2022]

Let $w \in L(M)$, and consider a computation in M on the input w:

$$C_M(w) : \mathbf{p}_0 w \models \mathbf{p}_1 w_1 \models \mathbf{p}_2 w_2 \models \ldots \models \mathbf{p}_m, \text{ where } \mathbf{p}_0 = s \text{ and } \mathbf{p}_m \in F,$$

$$E(C_M(w)) = \{i \geq 1 \mid \mathbf{p}_{i-1} w_{i-1} \circlearrowright \mathbf{p}_i w_i\}.$$

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$$jc_M(w) = \begin{cases} \min\{card(E(C_M(w))) \mid C_M(w) \text{ is a computation of } M \text{ on } w\} \\ 0, \text{ if } w \notin L(M). \end{cases}$$

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The *jumping complexity* of an automaton M is

$$jc_M(n) = \max\{jc_M(w) \mid |w| = n\}.$$

$\mathrm{JUMP}(f(n))$ class of langs. accepted by $\circlearrowright_R$**DFA** with $jc_M(n) = \mathcal{O}(f(n))$.

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The *sweep complexity* of an automaton M is $sc_M(n)$ is the maximum number of sweeps M makes on processing inputs $w \in L(M)$ of length n.
$\mathrm{SWEEP}(f(n))$ class of langs. accepted by $\circlearrowright_R$**DFA** with $sc_M(n) = \mathcal{O}(f(n))$.

Sweep complexities for bounded jumps [Fazekas et al., 2022]

Proposition

For any integer $k \geq 0$, the sweep complexity of any $ROWJFA_k$ is $O(\log n)$.

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Theorem

For every $k > 1$, there exists a $ROWJFA_{k-1}$ w/ sweep complexity $\Omega(\log n)$.

Deficit states [Fazekas et al., 2022]

Proposition

If a $\circlearrowright_R$**DFA** *has superconstant jump complexity, then it has two cyclic, reachable and co-reachable states* $\mathbf{p}$ *and* $\mathbf{q}$ *such that state* $\mathbf{p}$ *is a-deficient, for some* $a \in \Sigma$*, state* $\mathbf{q}$ *is reachable from* $\mathbf{p}$ *and* $\mathbf{q}aw \models^* \mathbf{q}$*, for some* $w \in \Sigma^*$*.*

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But then... [Fazekas and Mercaș, 2023]

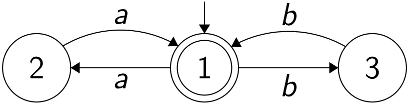

Figure: $L(\mathcal{B}) = \{w \in \{a, b\}^* \mid |w|_a \equiv 0 \bmod 2, |w|_b \equiv 0 \bmod 2\}$.

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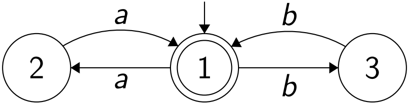

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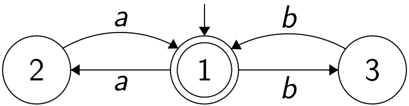

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The sweep complexity of $\mathcal{B}$ is $\Theta(\log n)$.

And we continue

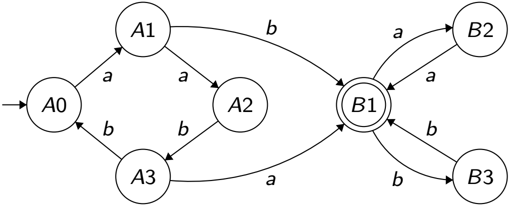

Figure: $L(\mathcal{C}) = \{w \in \{a,b\}^* \mid |w|_a \equiv 1 \mod 2, |w|_b \equiv 1 \mod 2\}$

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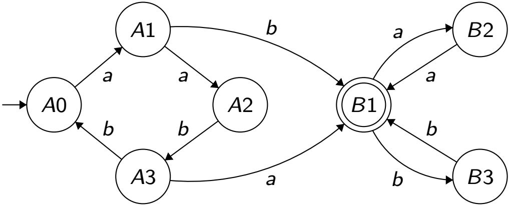

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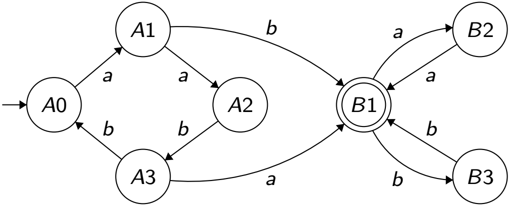

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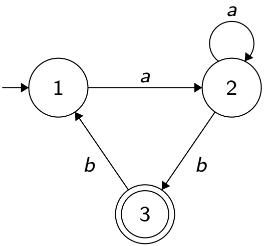
But **non-REG** languages also can have sublinear sweep complexity



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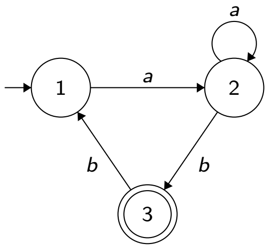

Lemma

The ROWJFA $\mathcal{D}$ accepts a non-regular language.

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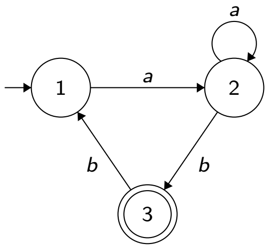

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$\mathcal{D}$ accepts $\varphi^n(ab)$ in $n+1$ sweeps, where $\varphi(a) = abab$, $\varphi(b) = b$.

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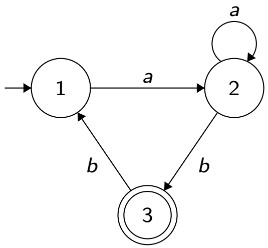

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Lemma

The sweep complexity of $\mathcal{D}$ is $\Theta(\log n)$.

Complexity classes separation

Theorem

$\mathit{SWEEP}(1) \subsetneq \mathit{SWEEP}(\log n)$

Lemma

Any automaton which accepts $L_{ab} = \{w \in \{a,b\}^* \mid |w|_a = |w|_b\}$ *has sweep complexity* $\Theta(n)$.

Theorem

For any $f : \mathbb{N} \Rightarrow \mathbb{N}$ *with* $f(n) \in o(n)$ *we have* $\mathit{SWEEP}(f(n)) \subsetneq \mathit{SWEEP}(n)$.

Answered several questions regarding sweep complexity [Fazekas et al., 2022]

1. Is the language of each machine with $\omega(1)$ complexity non-regular? NO.
2. Is there a machine with sweep complexity between constant and linear, that is, $\omega(1)$ and $o(n)$? YES.
3. Is there a *language* with sweep complexity between constant and linear, that is, all machines accepting it have superconstant complexity and at least one has sublinear? YES.
4. Is there an upper bound in terms of sweep complexity on machines accepting regular languages? NO.
5. Are machines less complex in the case of binary alphabets? NO.

What is next?

- do machines with arbitrary (constructible) sublinear complexity exist ($\Theta(\log^k n)$ and $\Theta(n^\epsilon)$, for constants $k > 1$ and $\epsilon < 1$)?
- logarithmic complexity can produce non-regular languages, but is it doable with less of this 'non-regular' resource?
- is it decidable given a machine or language and a function $f(n)$, whether the machine/language has $\Theta(f(n))$ sweep complexity?
- allow nondeterminism
- find sweet spot between extensions/restrictions and good closure props
- solve the open problems regarding equivalence and regularity

Thank you, VICTOR!

Questions?

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DFA for even and odd

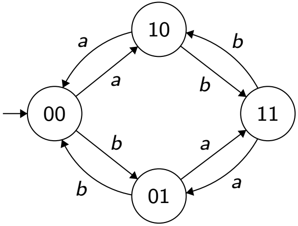

Figure: DFA accepting words with even (for **00** final state) or odd (for **11** final state) number of *a*'s and *b*'s.